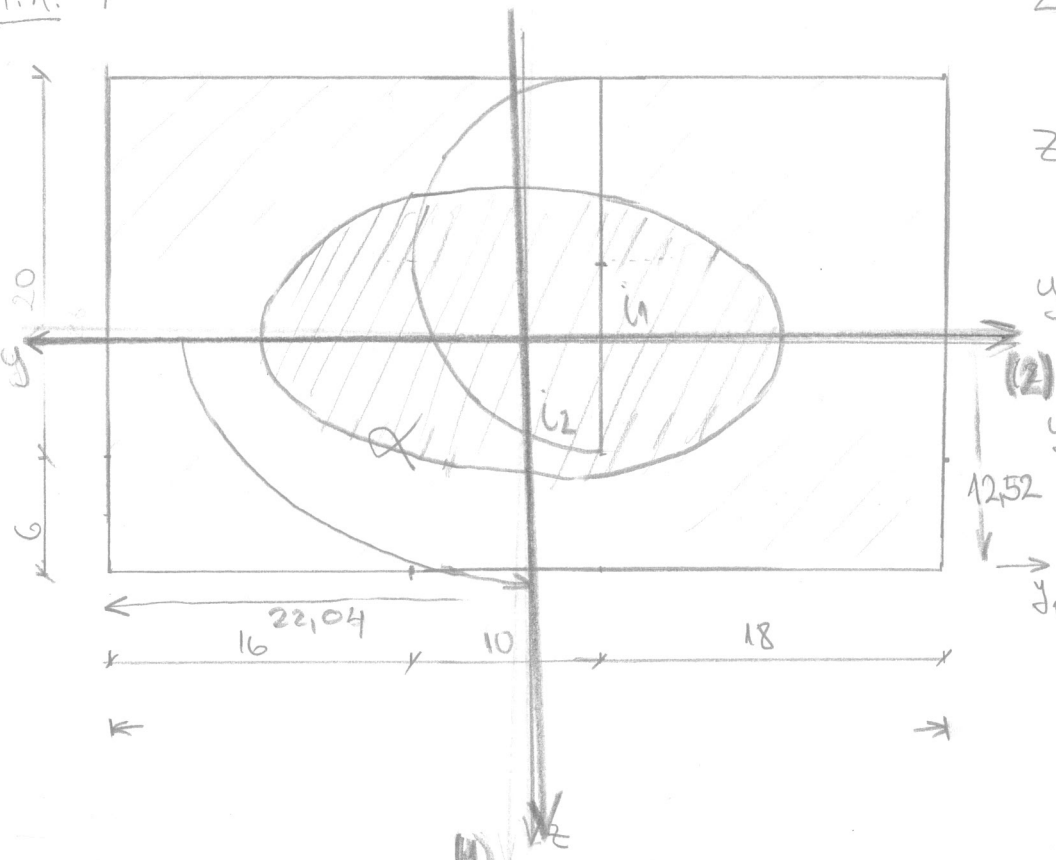


1.1.  $\uparrow z_1$



$$Z_T = \frac{44 \cdot 26 \cdot 13 - \frac{10^2 \pi}{4} \cdot 16}{44 \cdot 26 - \frac{10^2 \pi}{4}}$$

$$Z_T = \frac{12358,73}{986,92} = 12,52 \text{ cm}$$

$$y_T = \frac{44 \cdot 26 \cdot 22 - \frac{10^2 \pi}{4} \cdot 21,75}{986,92}$$

$$y_T = 22,04 \text{ cm}$$

$$I_y = \frac{1}{12} \cdot 44 \cdot 26^3 + 0,48^2 \cdot 1144 - \left( \frac{1}{8} 10^4 \pi + 3,48^2 \cdot 157,08 \right) = 58879,62 \text{ cm}^4$$

$$I_z = \frac{1}{12} \cdot 44^3 \cdot 26 + 0,04^2 \cdot 1144 - (0,10975 \cdot 10^4 + 0,29^2 \cdot 157,08) = 183456,45 \text{ cm}^4$$

$$I_{yz} = 0 + (-0,48)(+0,04) \cdot 1144 - (0 + (-3,48)(+0,29) \cdot 157,08) = 136,56 \text{ cm}^4$$

$$\tan 2\alpha = \frac{-2I_{yz}}{I_y - I_z} = \frac{-273,12}{-124577,45} = 0,002192371$$

III КВАДРАНТ

$$\alpha = \frac{1}{2} (180 + \arctan \rightarrow) = 90,06^\circ$$

$$I_{1,2} = 121168,035 \pm 62288,565$$

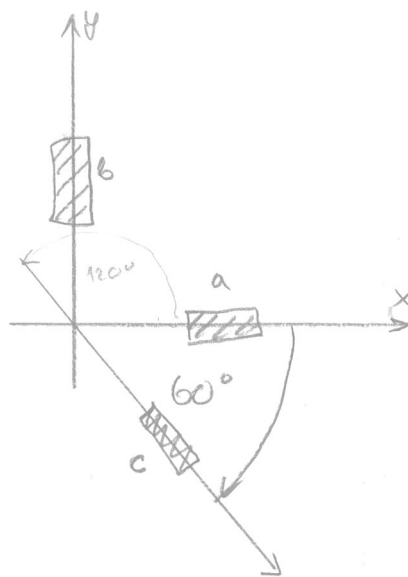
$$I_1 = 183456,6 \text{ cm}^4$$

$$I_2 = 58879,47 \text{ cm}^4$$

$$i_1 = \sqrt{\frac{I_1}{A}} = 13,63 \text{ cm}$$

$$i_2 = \sqrt{\frac{I_2}{A}} = 7,72 \text{ cm}$$

1.2  $E = 210 \text{ GPa}$   
 $G = 82 \text{ GPa}$   
 $\sigma_x = 25 \text{ MPa}$   
 $\sigma_y = 15 \text{ MPa}$   
 $\tau_{xy} = -10 \text{ MPa}$



a)  $\epsilon_x = \epsilon_a = \frac{1}{E} (\sigma_x - \nu \sigma_y)$

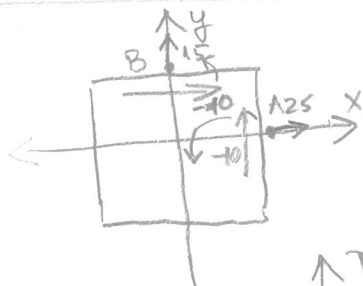
$\epsilon_y = \epsilon_b = \frac{1}{E} (\sigma_y - \nu \sigma_x)$

$\nu =$  ← УМН ФОРМУЛА У КЛУЗУ ПРЕКО G DA SE ИЗРАЗИ  
 ОДНАК 3. ПЕЊТО

$\epsilon_c = \epsilon_a \cos 120 + \epsilon_b \sin 120 + \gamma_{xy} \sin 120 \cos 120$

$\gamma_{xy} = \frac{1}{G} \cdot \tau_{xy}$

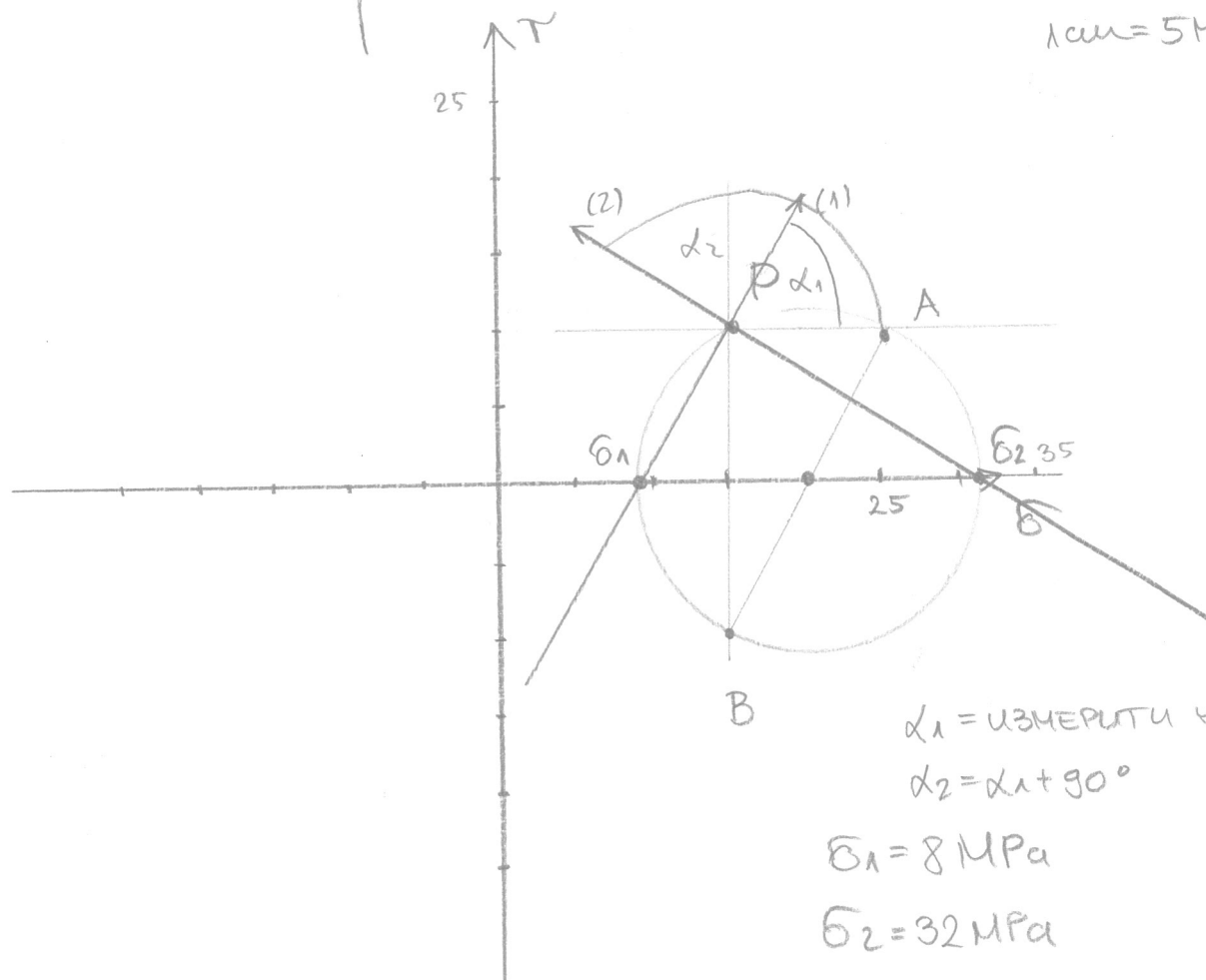
b)  $\begin{bmatrix} 25 & -10 \\ -10 & 15 \end{bmatrix}$



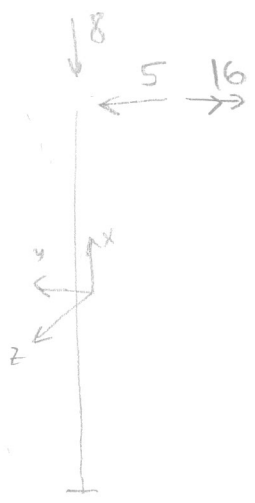
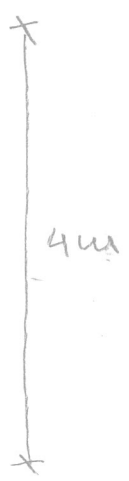
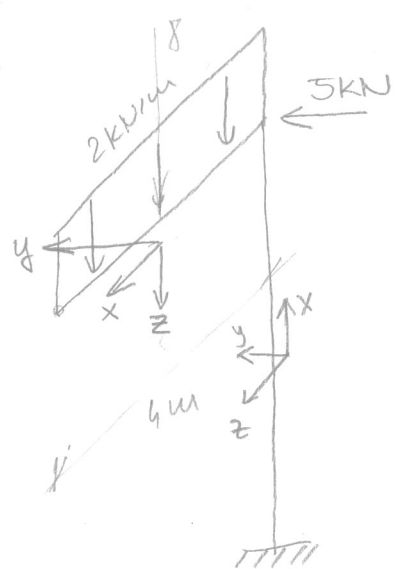
$A(25, 10)$

$B(15, -10)$

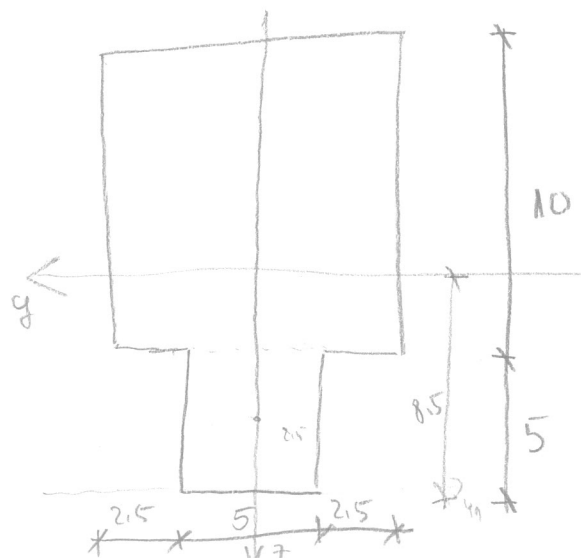
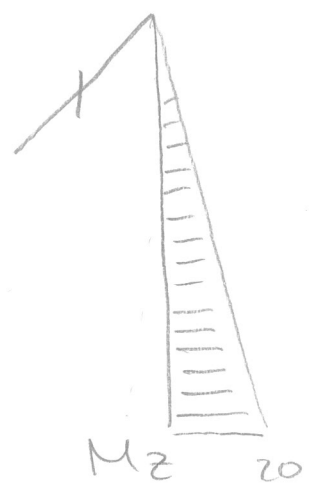
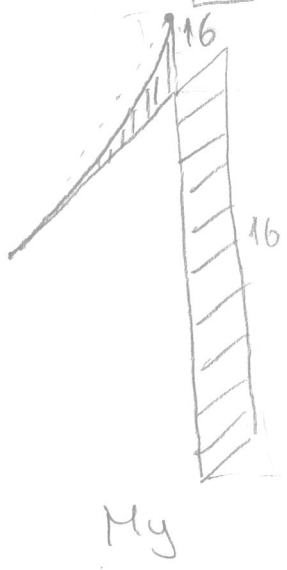
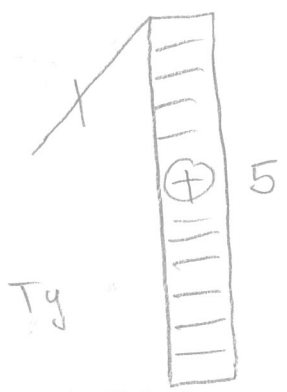
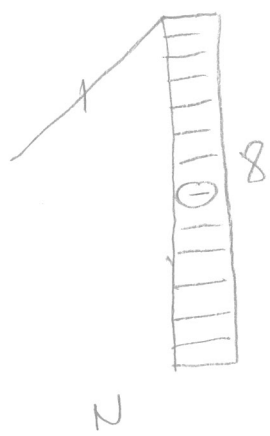
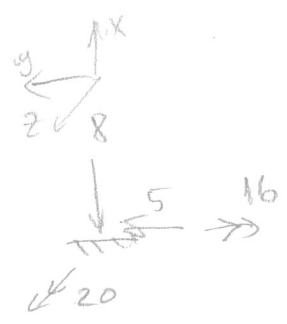
$1 \text{ cm} = 5 \text{ MPa}$



2.1



$x = \xi$   
 $y = \eta$   
 $z = \zeta$  (with  $\cos 2$ )



$$z_T = \frac{5 \cdot 5 \cdot 2,5 + 10 \cdot 10 \cdot 10}{125} = 8,5 \text{ cm}$$

$$I_y = \frac{1}{12} 5^4 + 6^2 \cdot 25 + \frac{1}{12} 10^4 + 1,5^2 \cdot 100$$

$$I_y = 2010,42 \text{ cm}^4$$

$$I_z = \frac{1}{12} 5^4 + \frac{1}{12} 10^4 = 885,42 \text{ cm}^4$$

НЕ ПРАВИЛЬН  
КООЗ (0,1,2)

$$I_y = 2010,42$$

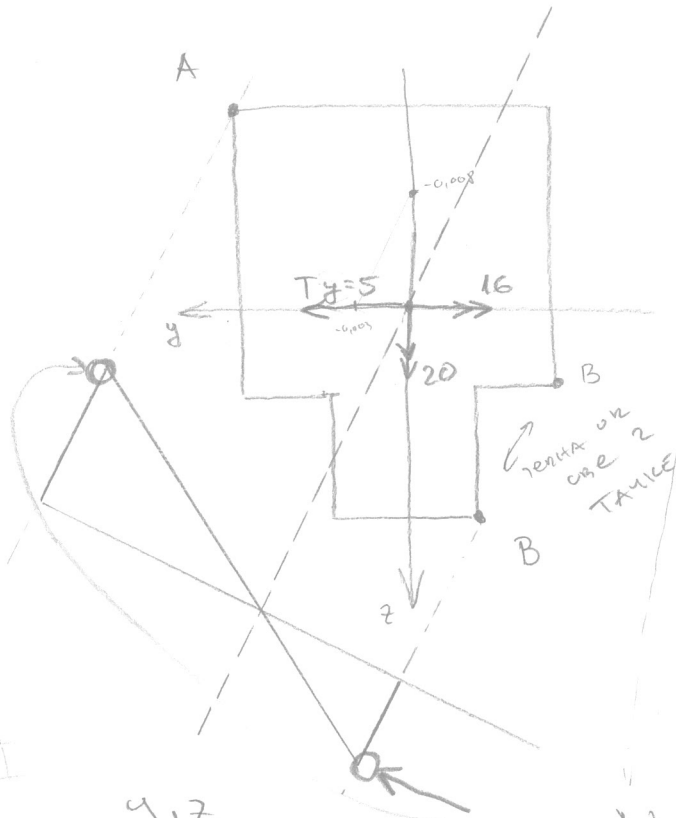
$$I_z = 885,42$$

$$A = 125$$

$$N = 8 \text{ kN}$$

$$M_y = 16 \text{ kNm}$$

$$M_z = 20 \text{ kNm}$$



$$O = \frac{8 \cdot 10^3}{125 \cdot 10^{-4}} - \frac{20 \cdot 10^3 \cdot y}{885,42 \cdot 10^{-8}} + \frac{(-16) \cdot 10^3 \cdot z}{2010,42 \cdot 10^{-8}}$$

$$O = 640000 - 2258815026y - 795849644,1z \quad / : 10^6$$

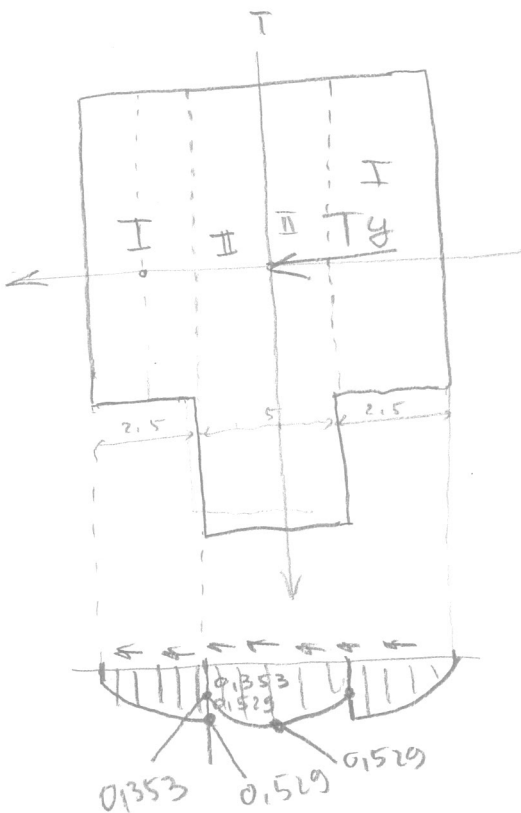
$$O = 0,64 - 2258,81y - 795,85z$$

$$0,64 - 2258,81y = 795,85z$$

y	0	0,00028
z	0,00008	0

у, z  
A (координате)  
B (координате)  
у, z

$$\sigma_x = \frac{N}{A} - \frac{M_z}{I_z} y + \frac{M_y}{I_y} z \rightarrow \text{решать и max скорости}$$



$$T_y = 5$$

$$I: T_{xy}^L = \frac{T_y \cdot S_z}{b(y) \cdot I_z} = \frac{5 \cdot 10^3 \cdot 93,75 \cdot 10^{-6}}{0,1 \cdot 885,42 \cdot 10^{-8}}$$

$$S_z^I = 2,5 \cdot 10 \cdot 3,75 = 93,75 \text{ cm}^3$$

$$T_{xy}^L = 0,529 \text{ MPa}$$

$$T_{xy}^D = \frac{5 \cdot 10^3 \cdot 93,75 \cdot 10^{-6}}{0,15 \cdot 885,42 \cdot 10^{-8}} =$$

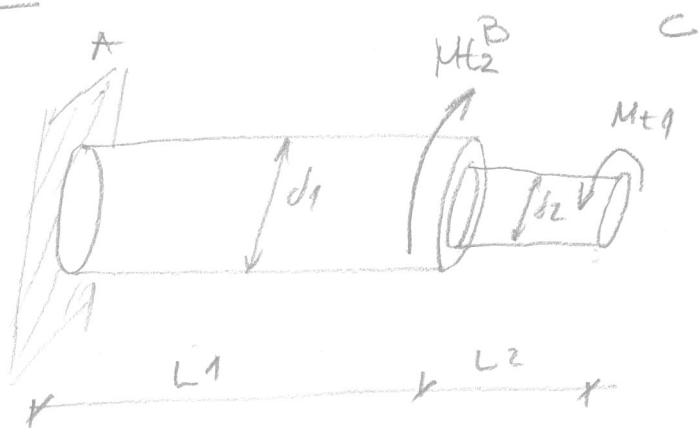
$$T_{xy}^D = 0,353 \text{ MPa}$$

$$T: T_{xy}^T = \frac{T_y \cdot S_z^T}{b(y) \cdot I_z} = \frac{5000 \cdot 140,625 \cdot 10^{-6}}{0,15 \cdot 885,42 \cdot 10^{-8}}$$

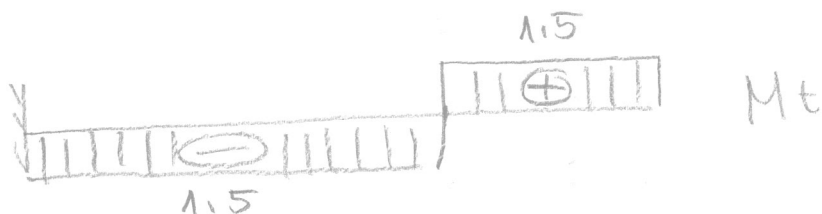
$$S_z^T = S_z^I + 2,5 \cdot 15 \cdot 1,25 = 140,625 \text{ cm}^3$$

$$T_{xy}^T = 0,529$$

2.2



$$\begin{aligned} d_2 &= 70 \text{ mm} \\ M_{t1} &= 1.5 \text{ kNm} \\ M_{t2} &= 2.5 \text{ kNm} \\ G &= 40 \text{ MPa} \\ L_1 &= 2.5 \text{ m} \\ L_2 &= 1.5 \text{ m} \end{aligned}$$



$$\tau_{\text{доп}} = 50 \text{ MPa}$$

$$M_{t1}^{AB} = 1.5 \text{ kNm}$$

$\tau$  - максимальная в кПа

$$\Delta \varphi = 0.02$$

$$\varphi_A = 0$$

$$\varphi_C < 0.02 \text{ рад}$$

$$\varphi = \frac{M_{t1}^{AB} \cdot L_1}{G I_{t1}} + \frac{M_{t2}^{BC} \cdot L_2}{G I_{t2}} \leftarrow \text{ВАЖНО ОБАКО?}$$

$d_1 = ?$   $\rightarrow$  ОБОБЩЕ НЕ СЕ НАТЪН  $d_2$

3.1

$q = 10 \text{ kN/m}$

25 kN

$w_D = ?$

$\varphi_a = ?$



37

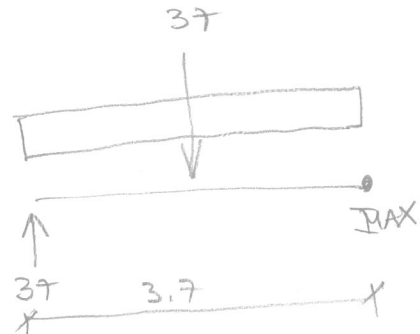
37

$\uparrow x_a = 37$

$\uparrow y_c = 48$

$$\sum M_a = 60 \cdot 3 - y_c \cdot 10 + 25 \cdot 12 \rightarrow y_c = 48$$

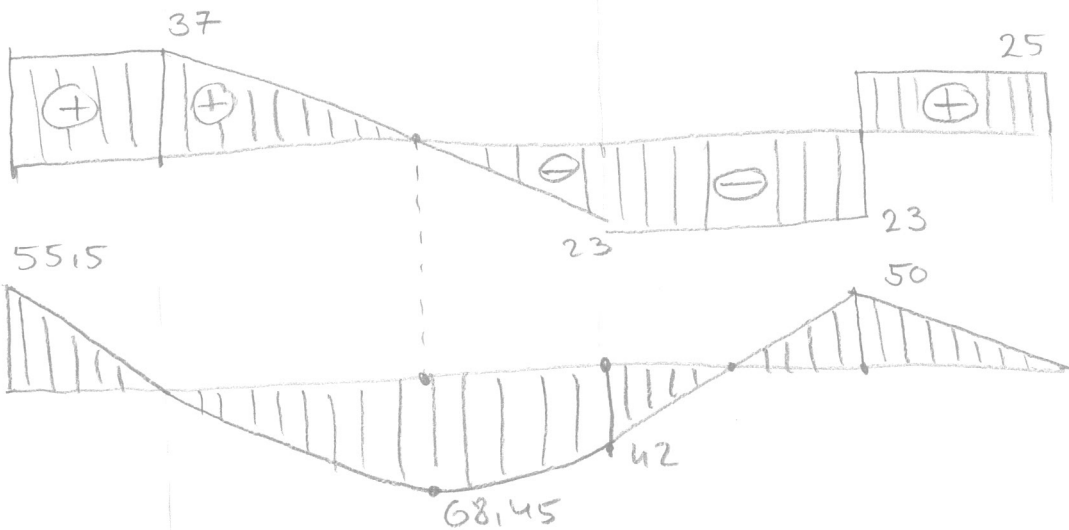
$$y_a = 37$$



$$60 : 6 = 37 : x$$

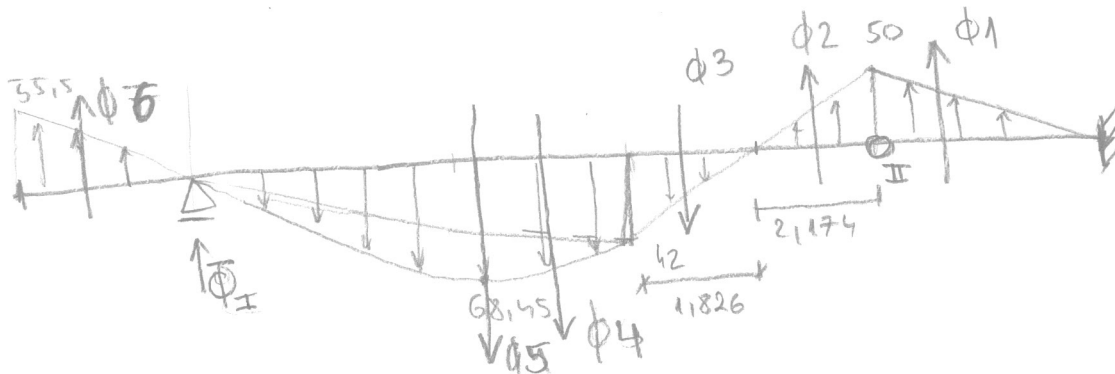
$$x = 3.7$$

$$y = 2.3$$



(T)

(M)



(phi)

$$92 : 4 = 50 : x$$

$$x = 2.174$$

$$\phi_1 = 50$$

$$\phi_4 = 126$$

$$\phi_2 = 54.35$$

$$\phi_5 = \frac{2}{3} \cdot 45 \cdot 6 = 180$$

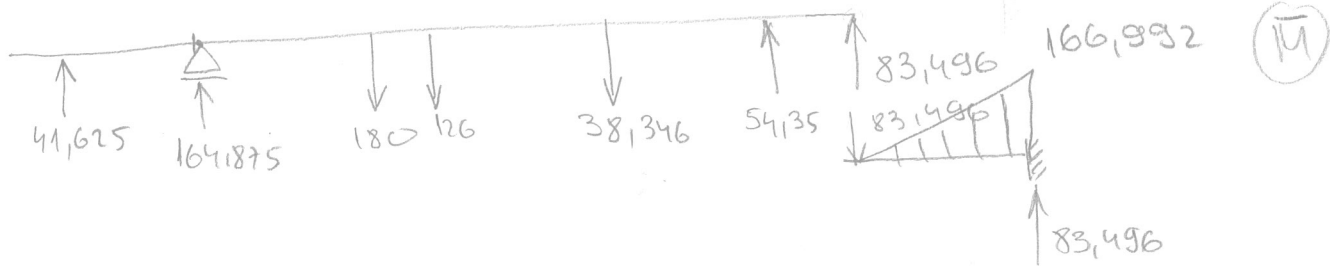
$$\phi_3 = 38.346$$

$$\phi_6 = 41.625$$

$$\sum M_{II} =$$

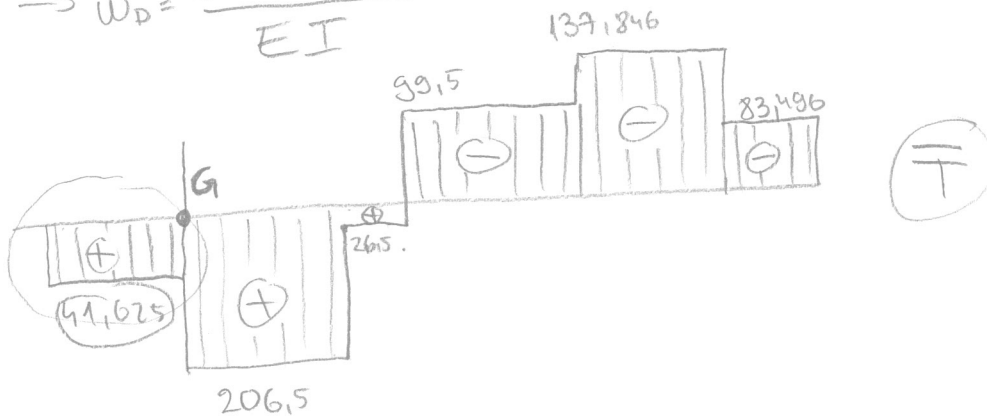
$$41,625 \cdot 11 + \phi_I \cdot 10 - 180 \cdot 7 - 126 \cdot 6 - 38,346 \cdot 3,391 + 54,35 \cdot 0,725 = 0$$

$$\phi_I = 164,875$$



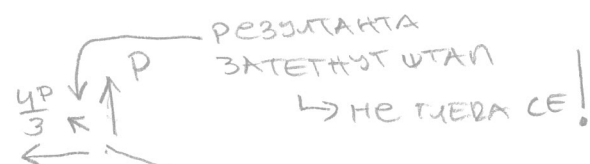
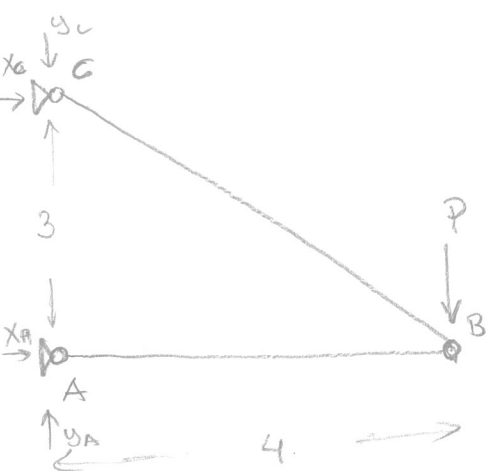
$$\overline{M}_y^D = 166,992$$

$$\rightarrow w_D = \frac{166,992 \cdot 10^2}{EI}$$



$$p_G^L = \frac{41,625 \cdot 10^3}{EI}$$

3.2



РЕЗУЛЬТАНТА ПРУТИЧУСТ УТАН

$$S_1 = \frac{4P}{3}$$

$$\sum M_A = 0$$

$$X_C \cdot 3 + P \cdot 4 = 0$$

$$3X_C = -4P$$

$$X_C = -\frac{4P}{3} = \frac{4P}{3}$$

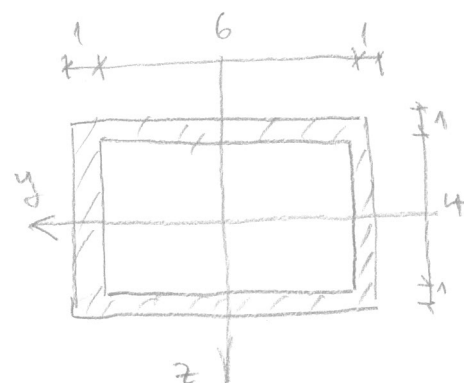
$$\sum M_B = 0$$

$$-\frac{4P}{3} \cdot 3 - y_C \cdot 4 = 0$$

$$-4P = 4y_C$$

$$y_C = -P$$

$$y_C = P$$



$$I_y = \frac{1}{12} (8 \cdot 6^3 - 6 \cdot 4^3) = 112 \text{ cm}^4$$

$$I_z = \frac{1}{12} (8^3 \cdot 6 - 6^3 \cdot 4) = 184 \text{ cm}^4$$

$$i_{\min} = \sqrt{\frac{I_y}{A} + \frac{112}{24}} = 2,16$$

$$\lambda = \frac{L}{i_{\min}} = \frac{400}{2,16} = 185,185$$

$$\sigma_{кр} = \min \begin{cases} \sigma_{кр}^E = \frac{\pi^2 \cdot E}{\lambda^2} = 60,44 \text{ MPa} \\ \sigma_{кр}^T = 310 - 1,14 \cdot \lambda = 98,89 \text{ MPa} \\ \sigma_T = 250 \text{ MPa} \end{cases}$$

$$\rightarrow \sigma_{кр} = 60,44 \text{ MPa}$$

$$P_{кр} = \sigma_{кр} \cdot A = 60,44 \cdot 10^6 \cdot 24 \cdot 10^{-4}$$

$$P_{кр} = 145,06 \text{ kN}$$

$$n_s \geq 3,5$$

$$n_s = \frac{P_{кр}}{S_1} = \frac{145,06}{\frac{4P}{3}} \geq 3,5$$

$$\frac{108,795}{P} \geq 3,5$$

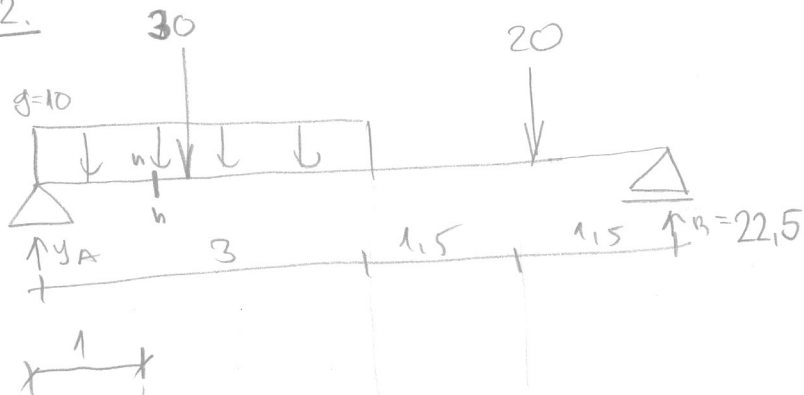
$$P \leq 31,08 \text{ kN}$$

$$E = 210 \text{ GPa}$$



1.1. МОМЕНТИ ИНЕРЦИЈЕ - ГРАЂЕНО У ЈАН 2011

1.2.

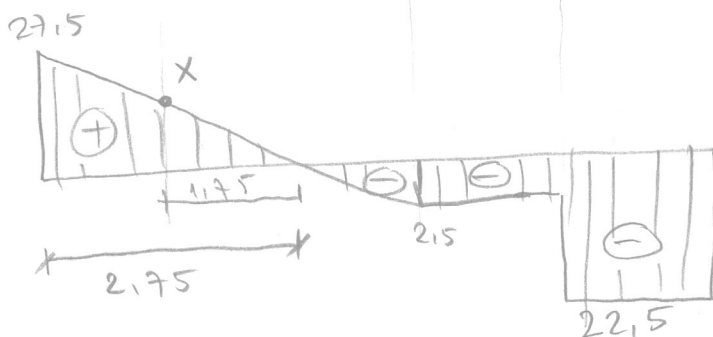


$$\sum M_A = 0$$

$$30 \cdot 1,5 + 20 \cdot 4,5 - y_B \cdot 6 = 0$$

$$y_B = 22,5$$

$$y_A = 27.5$$



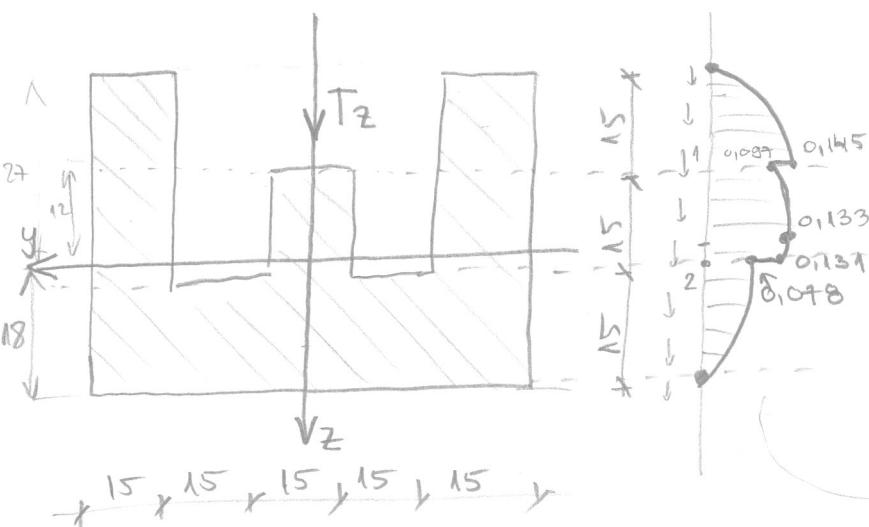
$$30:3 = 27,5:1$$

$$d = 2,75 \text{ cm}$$

$$27,5 : 2,75 = x : 1,75$$

$$X = 17,5 \text{ kN}$$

$$T_z = 17.5 \text{ kN}$$



$$Z_T = \frac{75 \cdot 15 \cdot 7,5 + 3 \cdot 15 \cdot 15 \cdot 22,5 + 2 \cdot 15 \cdot 15 \cdot 37,5}{75 \cdot 15 + 3 \cdot 15 \cdot 15 + 2 \cdot 15 \cdot 15}$$

$$2r = 18 \text{ cm}$$

$$I_y = \frac{1}{12} \cdot 75 \cdot 15^3 + (10,5^2 \cdot 1125) + 3 \left( \frac{1}{12} \cdot 15^4 + (4,5^2 \cdot 225) \right) + 2 \cdot \left( \frac{1}{12} \cdot 15^4 + (19,5^2 \cdot 225) \right)$$

$$I_y = 351000 \text{ cm}^4$$

$$S_1 = 2 \cdot 15 \cdot 15 \cdot 19,5 = 8775 \text{ cm}^3$$

$$T_{xz} = \frac{17.5 \cdot 10^3 \cdot 8775 \cdot 10^{-6}}{(2.0 \cdot 15) \cdot 351000 \cdot 10^{-8}} = 0.145 \text{ MPa}$$

$$\frac{10}{1 \times 2} = \frac{11}{(3.0, 15) - 11} = 0,094 \text{ MPa}$$

$$S_T = S_1 + 3 \cdot 12 \cdot 15 \cdot 6 = 12015$$

$$T_{xz} = \frac{17,5 \cdot 10^3 \cdot 12015 \cdot 10^{-6}}{(3 \cdot 0,15) \cdot 351000 \cdot 10^{-8}} = 0,13412$$

$$S_2 = 15.75 \cdot 10.5 = 11812.5 \text{ cm}^3$$

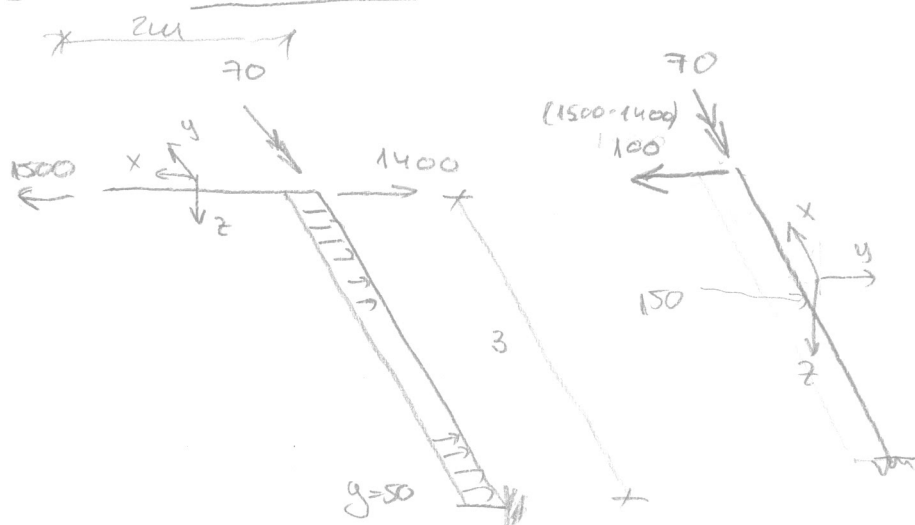
$$T_{xz}^{2D} = \frac{17.5 \cdot 10^3 \cdot 11812.5 \cdot 10^{-6}}{0.75 \cdot 351000 \cdot 10^{-8}}$$

$$T_{x_2}^{20} = 0,078 \text{ MPa}$$

$$T_{XZ} = \frac{17,5 \cdot 10^3 \cdot 11812,5 \cdot 10^{-6}}{(3 \cdot 0,15) \cdot 351000 \cdot 10^{-8}}$$

$$T_{xz} = 0,131 \text{ MPa}$$

2.1 ? → НУЖАМ СЛУШАТИ ДА МУ СЕ ПАЗИ ОБАКО!



$$I_y = \frac{1}{12} \cdot 45 \cdot 25^3 = 58593,75$$

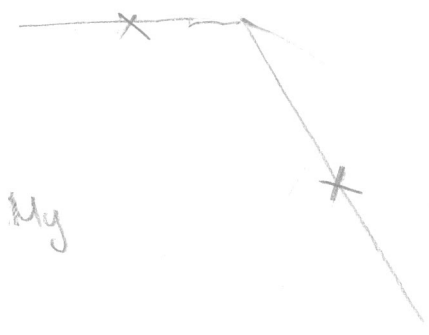
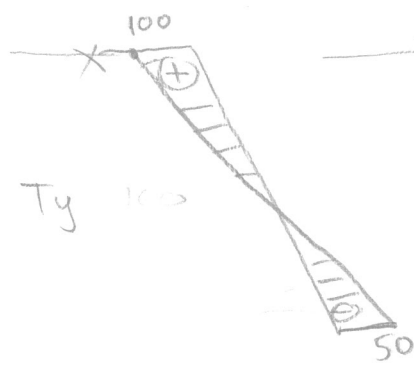
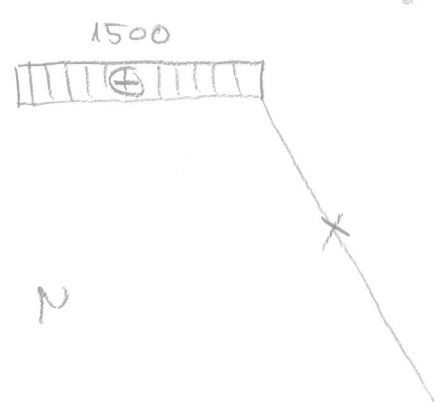
$$I_z = \frac{1}{12} \cdot 45^3 \cdot 25 = 189843,75$$

$$\frac{c}{b} = 1,8$$

$$1,5 - 0,231 = 1,269$$

$$1,8 - x = 0,24 \Rightarrow x = 1,56$$

$$2,0 - 0,246 = 1,754$$



8,88 MPa

$$\sigma_x = \frac{M_z}{I_z} y = \frac{75 \cdot 10^3}{189843,75} \cdot 10,225 = 8,88 \text{ MPa}$$

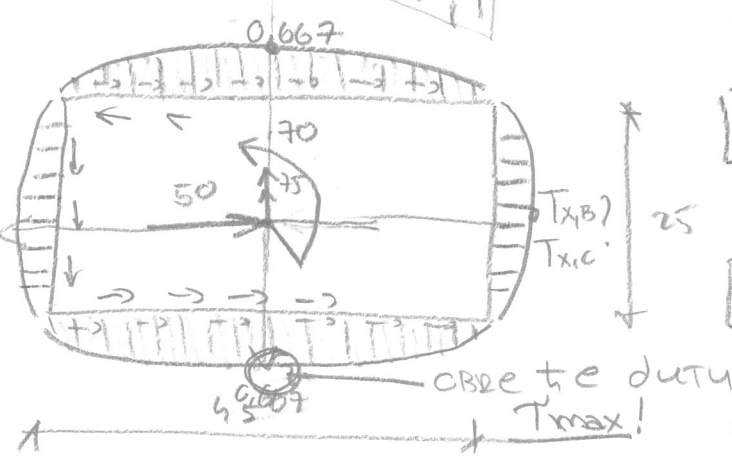
$$\tau_{xy} = \frac{3}{2} \frac{T_y}{A} = \frac{50 \cdot 10^3}{0,45 \cdot 0,25} \cdot 1,15 = 0,667 \text{ MPa}$$

$$\tau_{x,B} = \frac{M_t}{W_{tB}} = \frac{70 \cdot 10^3}{8750 \cdot 10^6} = 10,37 \text{ MPa ?}$$

$$W_{tB} = \beta \cdot b^2 \cdot c = 0,24 \cdot 25^2 \cdot 45 = 6750 \text{ cm}^3$$

$$\tau_{x,C} = \frac{M_t}{W_{tC}} = \frac{70 \cdot 10^3}{8240,62} = 8,49 \text{ MPa ?}$$

$$W_{tC} = \gamma \cdot b^2 \cdot c = 0,293 \cdot 25^2 \cdot 45 = 8240,62 \text{ cm}^3$$



$$\sigma_{x,max} = 8,88 \text{ MPa}$$

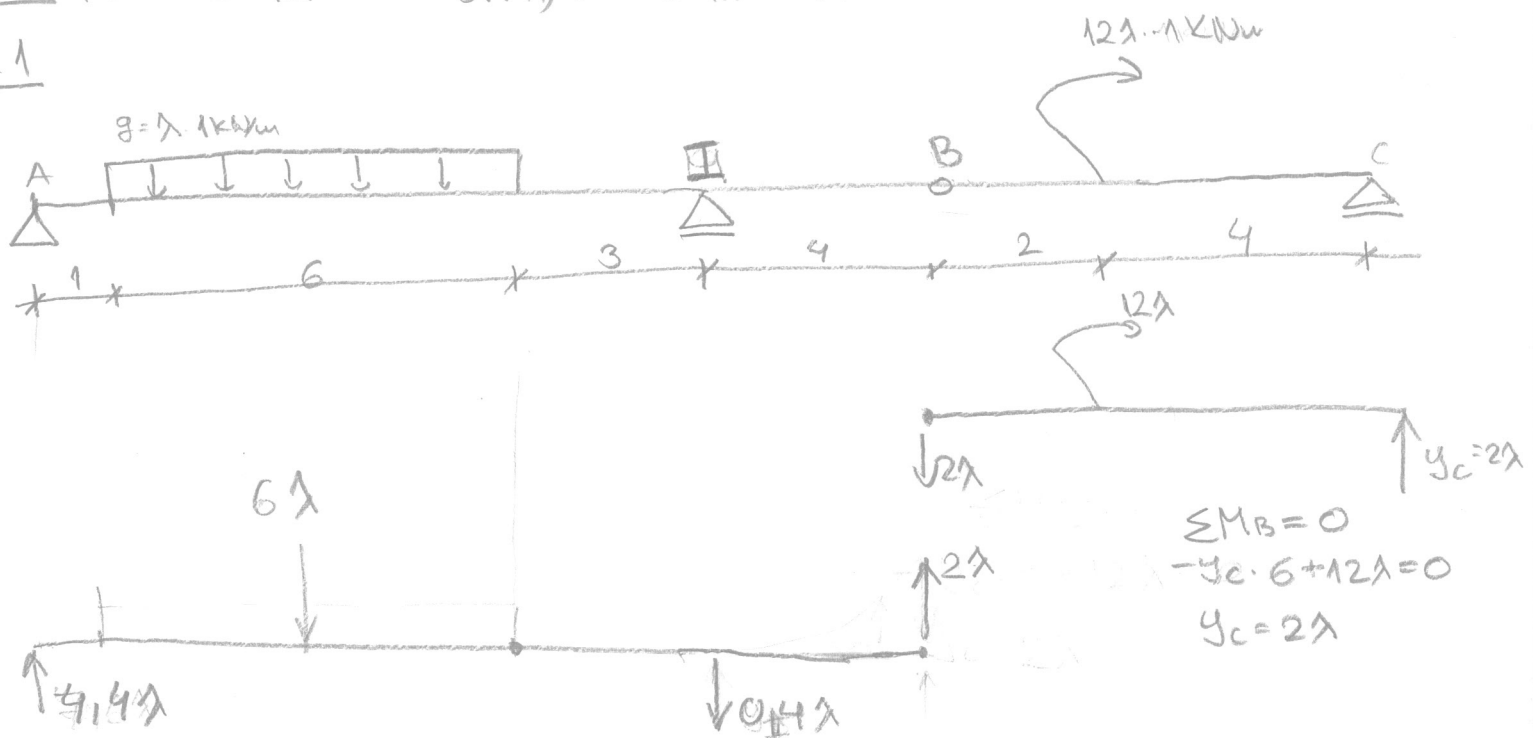
$$\tau_{x,max} = 10,37 \text{ MPa}$$

$\sigma_{max}$  - НАПРЯЖЕНИЕ СЕ СРЕДНЕГО БИ Х НАПРАВЛЕНИЮ У ИСТОЧ ТАЧКА

$\tau_{max}$  - НАПРЯЖЕНИЕ СЕ СРЕДНЕГО ТХ У ИСТОЧ ТАЧКА

## 2.2 МОРОВ КРЪГ - УРАЖЕН ЈАН 2011

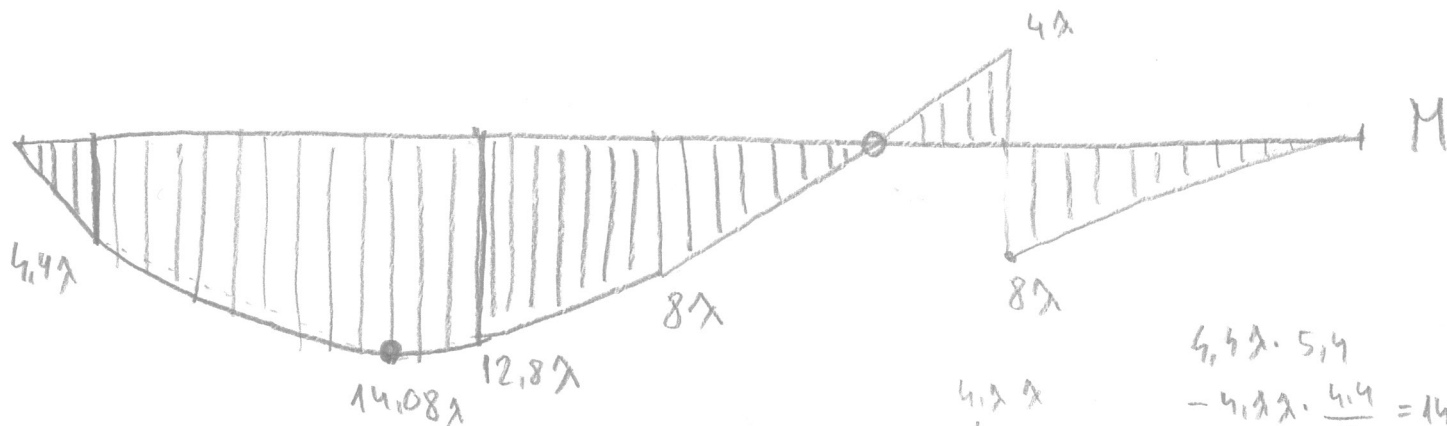
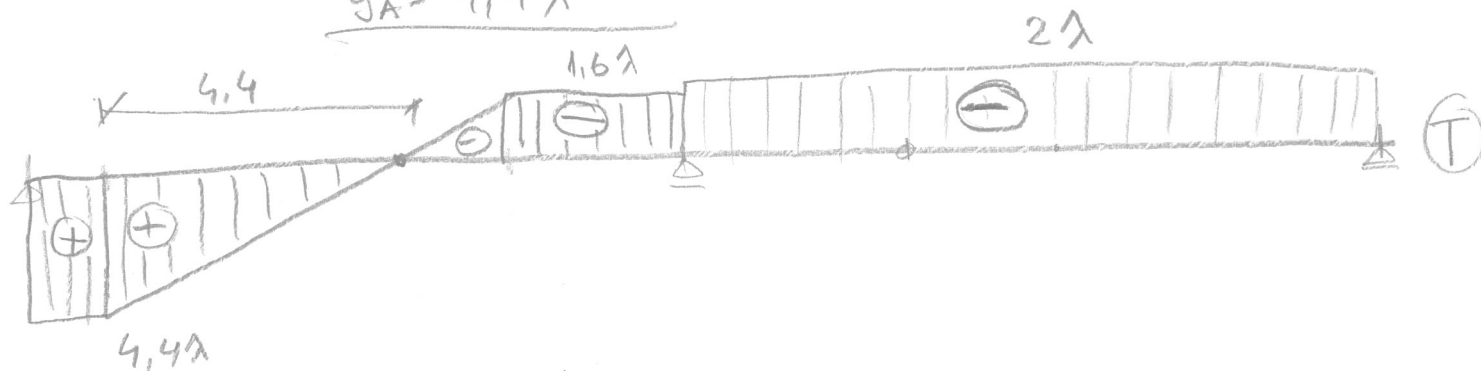
3.1



$$\begin{aligned} \sum M_B &= 0 \\ -y_C \cdot 6 + 12\lambda &= 0 \\ y_C &= 2\lambda \end{aligned}$$

$$\begin{aligned} \sum M_A &= 0 \\ 6\lambda \cdot 4 - y_B \cdot 10 - 2\lambda \cdot 14 &= 0 \\ -4\lambda &= 10y_B \rightarrow y_B = -0.4\lambda \end{aligned}$$

$$y_A = 4.4\lambda$$

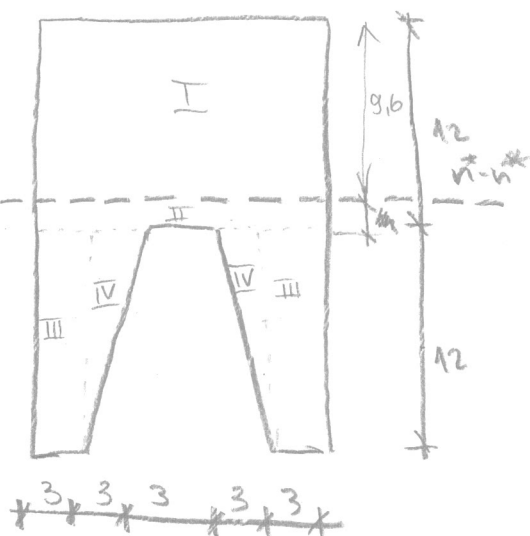


$$\begin{aligned} 4.4\lambda \cdot 5.4 \\ - 4.4\lambda \cdot \frac{4.4}{2} &= 14.08\lambda \end{aligned}$$



AB

15



$$M_{AB}^* = 14,08 \lambda$$

$$H^* = 6 \cdot W^*$$

$$A^* = 12 \cdot 15 + 2 \cdot 3 \cdot 12 + 2 \cdot \frac{1}{2} \cdot 3 \cdot 12 = 288 \text{ cm}^2$$

$$\frac{A}{2} = 144 \text{ cm}^2$$

$$A_I = 15 \cdot x = 144 \quad x = 9,6 \text{ cm}$$

$$S_I^* = 15 \cdot 9,6 \cdot 4,8 = 835,2 \text{ cm}^3$$

$$S_{II}^* = 2,4 \cdot 15 \cdot 1,2 = 43,2 \text{ cm}^3$$

$$S_{III}^* = 2 \cdot 3 \cdot 12 \cdot 8,4 = 604,8 \text{ cm}^3$$

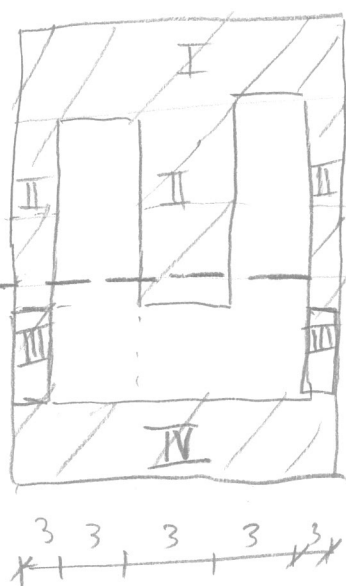
$$S_{IV}^* = 2 \cdot \frac{1}{2} \cdot 3 \cdot 12 \cdot 6,4 = 230,4 \text{ cm}^3$$

$$W^* = \sum S^* = 1713,6 \text{ cm}^3$$

$$14,08 \lambda = 24 \cdot 10^6 \cdot 1713,6 \cdot 10^{-6}$$

$$14,08 \lambda = 41126,4$$

$$14,08 \cdot 10^3 \lambda = 41126,4 \rightarrow \boxed{\lambda_1 = 2,92}$$

BC

$$M_{BC}^* = 8 \lambda$$

$$A = 15 \cdot 20 - 2 \cdot 3 \cdot 12 - 3 \cdot 4 = 216 \text{ cm}^2$$

$$\frac{A}{2} = 108 \text{ cm}^2$$

$$4 \cdot 15 = 60$$

$$- 108$$

$$48$$

$$4 \cdot 3 \cdot 2 = 24$$

$$- 48$$

$$24$$

$$3 \cdot 3 \cdot x = 24$$

$$x = 2,67$$

$$S_I = 4 \cdot 15 \cdot 7,33 = 439,8$$

$$S_{II} = 3 \cdot 3 \cdot 5,33 \cdot 2,665 = 127,84$$

$$S_{III} = 2 \cdot 3 \cdot 6,67 \cdot 3,335 = 133,47$$

$$S_{IV} = 4 \cdot 15 \cdot 8,67 = 520,2$$

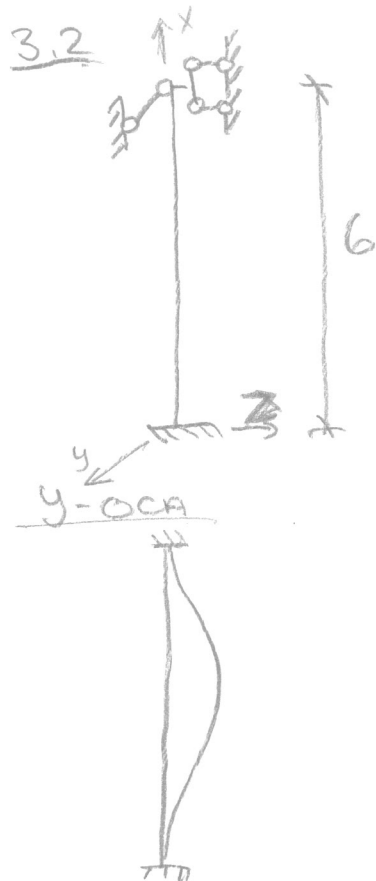
$$W^* = 1221,31 \text{ cm}^3$$

$$8 \lambda = 24 \cdot 10^6 \cdot 1221,31 \cdot 10^{-6}$$

$$8 \cdot 10^3 \lambda = 29311,44$$

$$\boxed{\lambda_2 = 3,66}$$

ГОБОЖЕНО  $\lambda_1$ !



$$l_0 = 0,5l = 3\text{m}$$

$$\lambda = \frac{l_0}{i_{\min}} = \frac{300}{5,177} = 57,99$$

$$\sigma_{kr} = \min \begin{cases} \sigma_{kr}^E = \frac{\pi^2 E}{\lambda^2} = 766\text{MPa} \\ \sigma_{kr}^T = 248,03\text{MPa} \\ \sigma_T = 240\text{MPa} \end{cases}$$

$$\sigma_{kr} = 240\text{MPa}$$

$$P_{kr} = \sigma_{kr} \cdot A =$$

$$240 \cdot 10^6 \cdot 240 \cdot 10^{-4} =$$

$$= 5760\text{KN}$$



$$l_0 = 0,7l = 4,2\text{m}$$

$$\lambda = \frac{420}{5,177} = 81,19$$

$$\sigma_{kr} = \min \begin{cases} \sigma_{kr}^E = 391,18\text{MPa} \\ \sigma_{kr}^T = 223,23\text{MPa} \\ \sigma_T = 240 \end{cases}$$

$$\sigma_{kr} = 223,23\text{MPa}$$

$$P_{kr} = 223,23 \cdot 10^6 \cdot 240 \cdot 10^{-4}$$

$$= 5357,52\text{KN}$$

↑  
выбрано  $P_{kr}$

$$I_y = 8000\text{cm}^4$$

$$I_z = 15000\text{cm}^4$$

$$A = 240\text{cm}^2$$

$$E = 210\text{GPa}$$

$$\sigma_T = 240\text{MPa}$$

$$\sigma_{kr}^T = 310 - 1,192\text{MPa}$$

$$i_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{8000}{240}} = 5,77\text{cm}$$

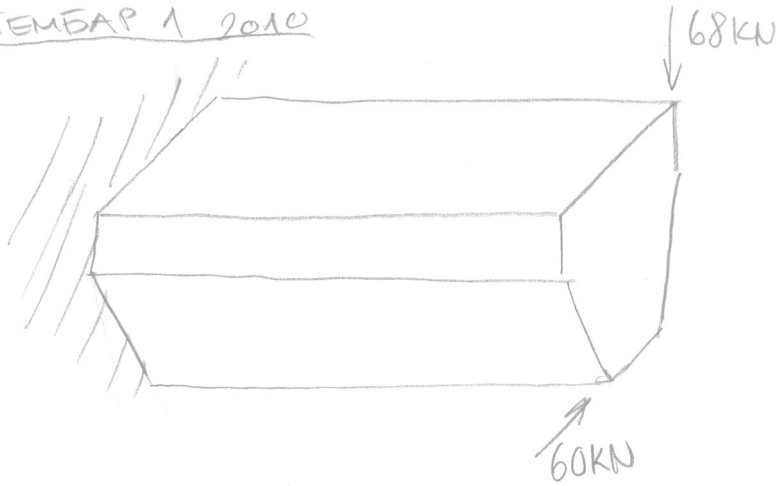
$$i_z = \dots = 7,91\text{cm}$$

$$i_{\min} = 5,77\text{cm}$$

↑  
БАЗОВАЯ ЧИСЛО  
ОБЪЕКТОВ

СЕНТЕМБАР 1 2010

1.1

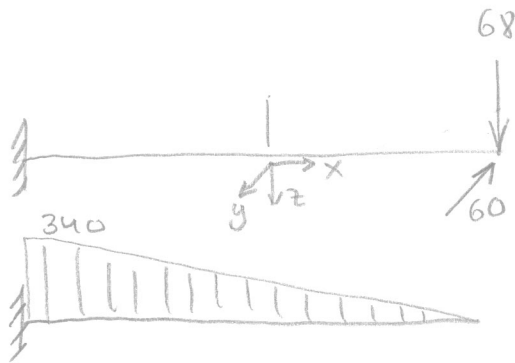
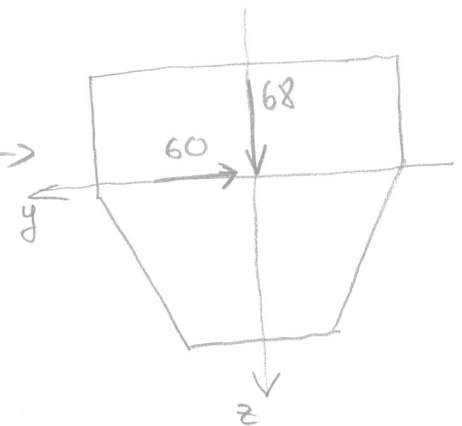
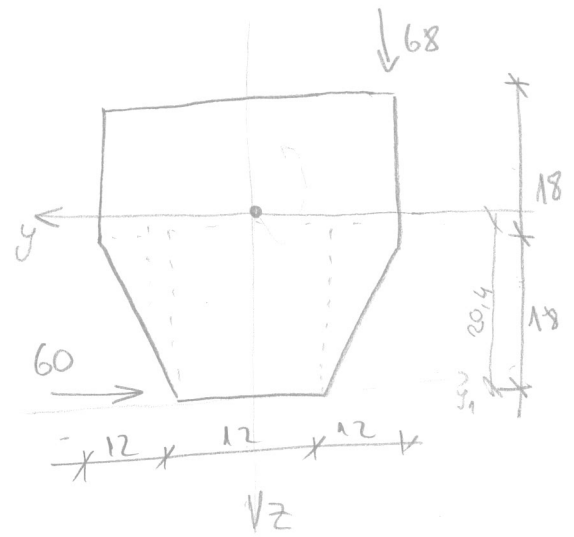


$$z_T = \frac{12 \cdot 18 \cdot 9 + 2 \cdot \frac{1}{2} \cdot 12 \cdot 18 \cdot 12 + 18 \cdot 36 \cdot 27}{1080} = 20,4 \text{ см}$$

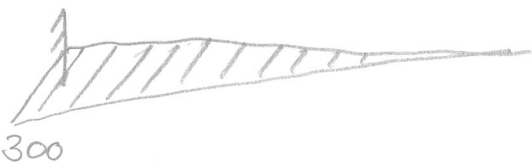
$$M_{t1} = 60 \cdot 20,4 = 1224 \text{ кНсм}$$

$$M_{t2} = 68 \cdot 18 = 1224 \text{ кНсм}$$

попечтавады се →



$M_y (M_y, T_z)$



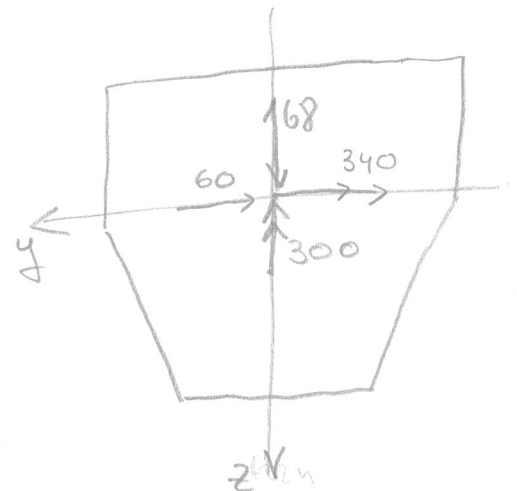
$M_z (M_z, T_y)$



$$I_y = \frac{1}{12} 36^4 + (2 \cdot 4^2 \cdot 1296) - 2 \cdot \left( \frac{1}{36} \cdot 12 \cdot 18^3 + (8 \cdot 4^2 \cdot 108) \right) = 128304 \text{ см}^4$$

$$I_z = \frac{1}{12} 36^4 - 2 \cdot \left( \frac{1}{36} \cdot 12^3 \cdot 18 + (10^2 \cdot 108) \right) = 116640 \text{ см}^4$$

$$I_{yz} = 0$$



$$\sigma_x = \frac{N}{A} - \frac{M_z}{I_z} y + \frac{M_y}{I_y} z$$

$$0 = \frac{-300 \cdot 10^3}{128304 \cdot 10^8} y + \frac{-340 \cdot 10^3}{116640 \cdot 10^8} z$$

$$0 = 233819678,3 y - 291495198,9 z \quad / : 291495198,9$$

$$0,802 y = z$$

$$I_y = 128304 \text{ cm}^4$$

$$I_z = 116640 \text{ cm}^4$$

$$M_y = -340 \text{ kNm}$$

$$M_z = -300 \text{ kNm}$$

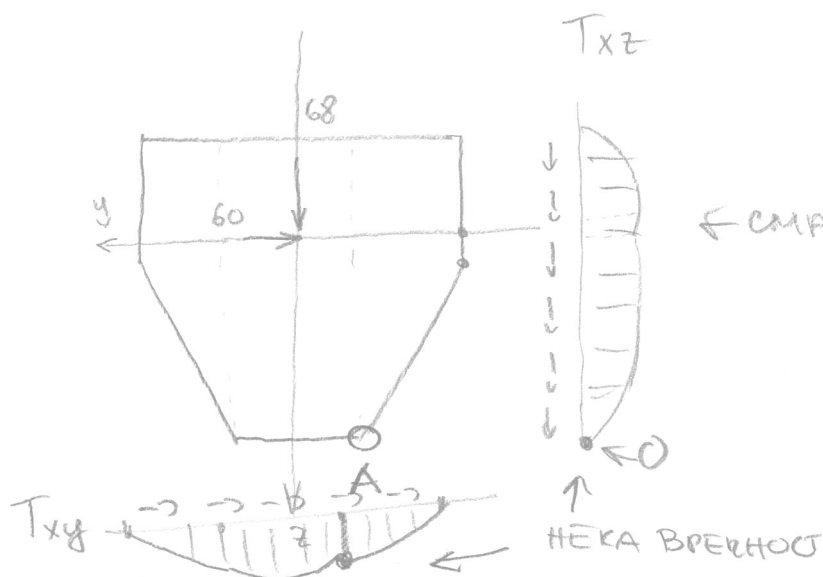
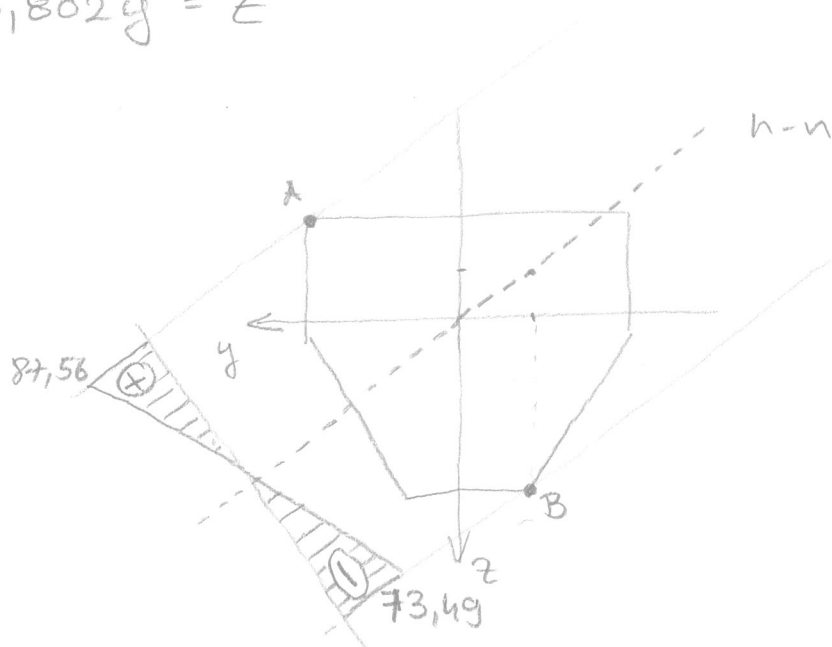
y	0	1	-1
z	0	0,802	-0,802

$$A(0,18; -0,156)$$

$$B(-0,06; 0,204)$$

$$A: \sigma_x = 87,56 \text{ MPa}$$

$$B: \sigma_x = -73,49 \text{ MPa}$$



← СМАРА, УМА НА ПРЕТХОЗНИМ НАПРИЛИМА

↑ НЕКА ВРЕЗНОСТ

$$S_z^I = 15,6 \cdot 36 \cdot 7,8 = 4380,48 \text{ cm}^3$$

$$T_{xz}^I = \dots$$

АНАЛИЗА СТАЊА У ТАЧКИ А:

$$S_A = \begin{bmatrix} \sigma_A & T_{xy} & T_{xz} \\ T_{yx} & 0 & 0 \\ T_{zx} & 0 & 0 \end{bmatrix}$$

← Б В ОА ГОРЊЕГ ЦРТЕЖА)  
 ↑ О  
 СА ДОЊЕГ ЦРТЕЖА

1.2

$$\vec{n} = \frac{1}{\sqrt{2}} \vec{j} + \frac{1}{\sqrt{2}} \vec{k}$$

$$S = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix} \text{ MPa}$$

$$\sigma_x^{(n)} = \epsilon_x \cdot n_x + \tau_{xy} n_y + \tau_{zx} n_z$$

$$= 1 \cdot 0 + 2 \cdot \frac{1}{\sqrt{2}} + 3 \cdot \frac{1}{\sqrt{2}} = \frac{5}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$$

$$\sigma_y^{(n)} = \tau_{yx} n_x + \epsilon_y n_y + \tau_{yz} n_z$$

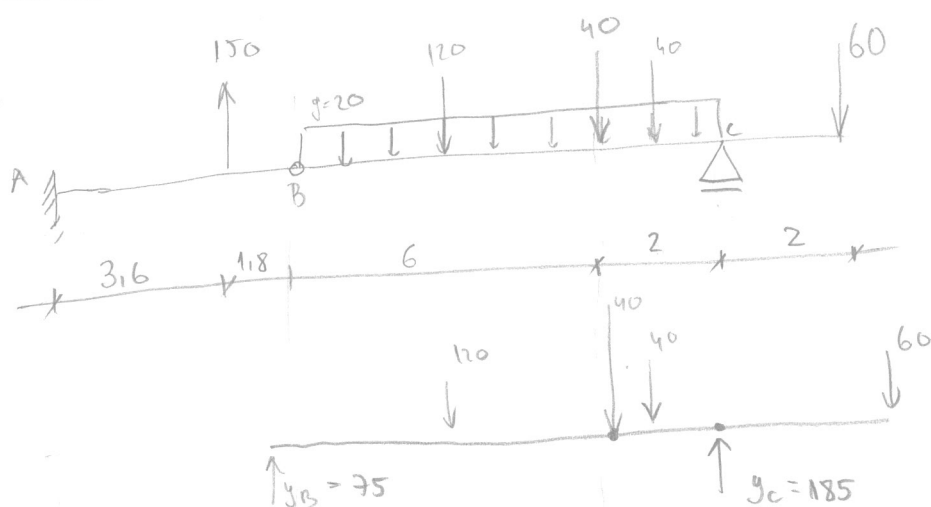
$$= 2 \cdot 0 + 4 \cdot \frac{1}{\sqrt{2}} + 6 \cdot \frac{1}{\sqrt{2}} = \frac{10\sqrt{2}}{2} = 5\sqrt{2}$$

$$\sigma_z^{(n)} = \tau_{zx} n_x + \tau_{zy} n_y + \epsilon_z n_z$$

$$= 3 \cdot 0 + 6 \cdot \frac{1}{\sqrt{2}} + 9 \cdot \frac{1}{\sqrt{2}} = \frac{15\sqrt{2}}{2}$$

$$|\vec{\sigma}| = \sqrt{\left(\frac{5\sqrt{2}}{2}\right)^2 + (5\sqrt{2})^2 + \left(\frac{15\sqrt{2}}{2}\right)^2} = 13,23$$

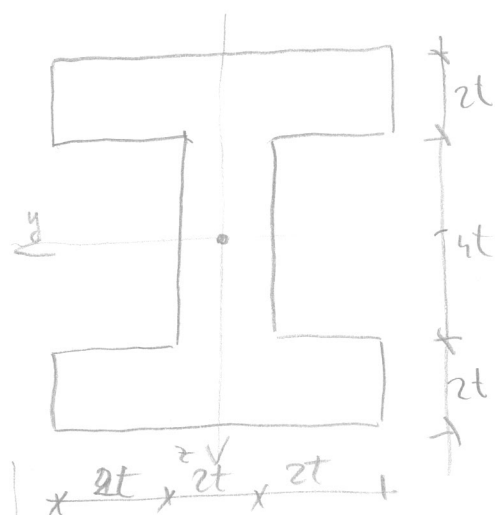
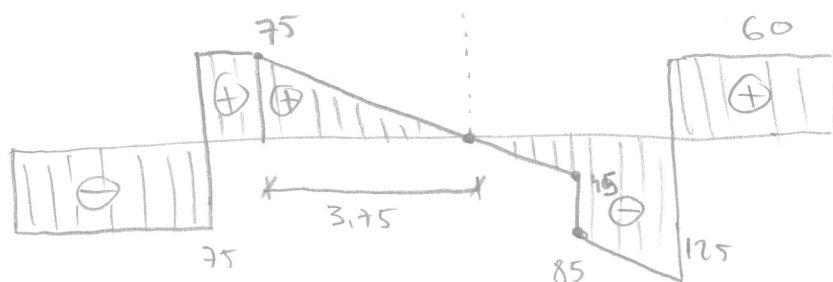
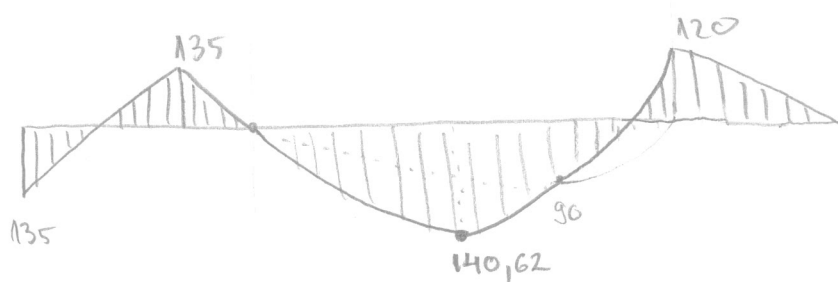
2.1



$$\sum M_B = 120 \cdot 3 + 40 \cdot 6 + 40 \cdot 7 - y_C \cdot 8 + 60 \cdot 10$$

$$\Rightarrow y_C = 185$$

$$y_B = 75$$



$$I_y = \frac{1}{12} 6t \cdot (8t^3) - 2 \cdot \frac{1}{12} \cdot 2t \cdot 4t^3$$

$$= 256t^4 - 21,33t^4$$

$$= 234,67t^4$$

$$W = \frac{I_y}{z_{max}} = \frac{234,67t^4}{4t}$$

$$W = 58,67t^3$$

$$\sigma_{max} = \frac{M_{max}}{W}$$

$$M_{max} = 140,62 \text{ kN}$$

$$\sigma_{max} < \sigma_{dop} = 20 \text{ MPa}$$

$$20 \cdot 10^6 > \frac{140,62 \cdot 10^3}{58,67t^3}$$

$$t > 0,049 \text{ m}$$

$$t_{usv} = 5 \text{ cm}$$

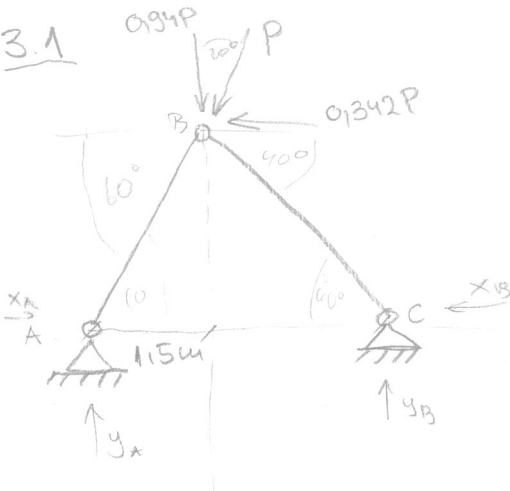
↑ prema  $\sigma_{dop}$





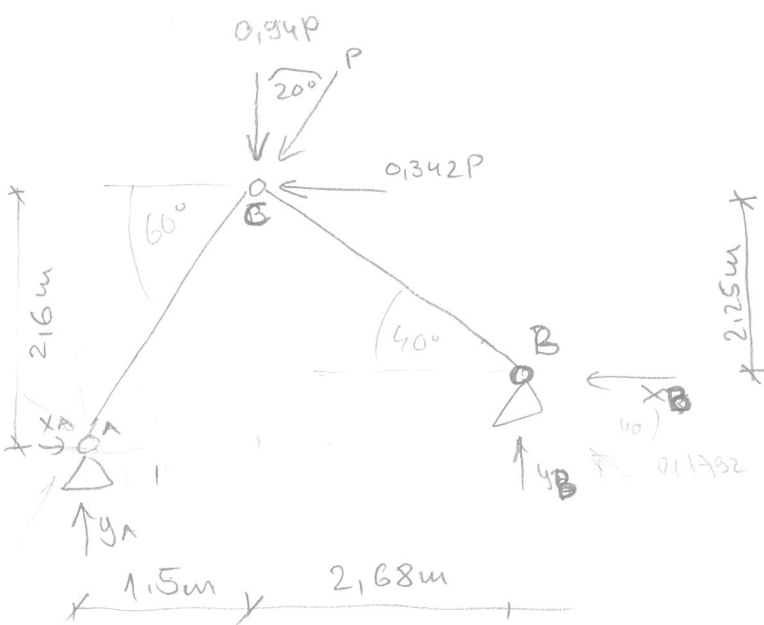
$t > 4.9 \text{ cm}$

3.1

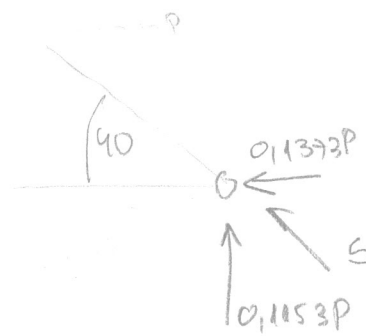
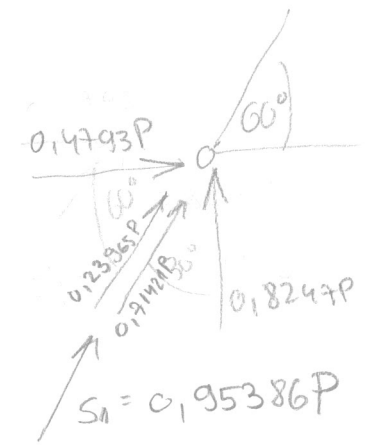


$L_{AB} = 3 \text{ m}$   
 $L_{BC} = 3.5 \text{ m}$

$$\begin{aligned} \sum M_A &= 0.934P \cdot 1.15 - 0.342P \cdot 2.16 \\ &- Y_B \cdot 4.18 - X_B \cdot 0.135 = 0 \\ 1.41P - 0.8892P \\ &- 4.18Y_B - 0.35X_B = 0 \\ -4.18Y_B - 0.35X_B &= -0.5208P \\ 4.18Y_B + 0.35X_B &= 0.5208P \end{aligned}$$



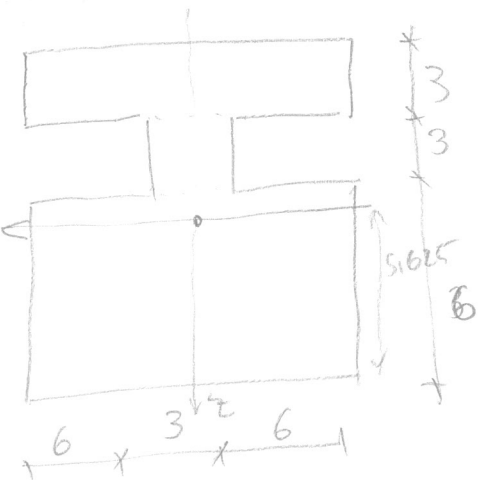
$$\begin{aligned} \sum M_E &= -Y_B \cdot 2.68 + 2.25X_B = 0 \\ -Y_B \cdot 2.68 &= -2.25X_B \\ X_B &= 1.191Y_B \\ 4.1Y_B + 0.4169Y_B &= 0.5208P \\ 4.5169Y_B &= 0.5208P \\ Y_B &= 0.1153P \\ X_B &= 0.1373P \end{aligned}$$



$$\begin{aligned} X_A &= 0.4793P \\ Y_A &= 0.8247P \end{aligned}$$

$$\begin{aligned} S_1 &= 0.17929P \\ 0.1153P \end{aligned}$$

ОБА ШТАПА ПРИТИСНУТА!



$$Z_T = \frac{3 \cdot 15 \cdot 10.5 + 3 \cdot 3 \cdot 7.5 + 6 \cdot 15 \cdot 3}{144}$$

$$Z_T = 5.625$$

$$I_y = \frac{1}{12} 15 \cdot 12^3 + 0.375^2 \cdot 180 - 2 \cdot \left( \frac{1}{12} 6 \cdot 3^3 + 1.875^2 \cdot 18 \right)$$

$$I_y = 2031.75 \text{ cm}^4$$

$$I_z = \frac{1}{12} 15^3 \cdot 12 - 2 \cdot \left( \frac{1}{12} 6^3 \cdot 3 + 4.5^2 \cdot 18 \right)$$

$$I_z = 2538 \text{ cm}^4$$

$$i_{min} = \sqrt{\frac{I_y}{A}} = 3.756$$

S1

$$\lambda = \frac{L_{AB}}{i_{min}} = \frac{300}{3,756} = 79,87$$

$$\sigma_{кр} = \min \begin{cases} \sigma_{кр}^E = \frac{\pi^2 E}{\lambda^2} = 324,90 \text{ МПа} \\ \sigma_{кр}^T = 310 \cdot 1,14 \lambda = 218,95 \text{ МПа} \\ \sigma_T = 260 \text{ МПа} \end{cases}$$

$$\sigma_{кр} = 218,29 \text{ МПа}$$

$$P_{кр} = 218,29 \cdot 10^6 \cdot 114 \cdot 10^{-4} = 3143,38 \text{ КН}$$

$$n_s \geq 3,5$$

$$\frac{P_{кр}}{S_1} \geq 3,5 \quad \frac{3143,38}{0,95386P} \geq 3,5$$

$$P_1 \leq 942,15 \text{ КН}$$

УСЛОВЕНО  $P_1$  ЗА КРИТИЧНУ СИЛУ

S2

$$\lambda = \frac{350}{3,756} = 93,184$$

$$\sigma_{кр} = \min \begin{cases} \sigma_{кр}^E = 249,276 \text{ МПа} \\ \sigma_{кр}^T = 203,77 \text{ МПа} \\ \sigma_T = 260 \text{ МПа} \end{cases}$$

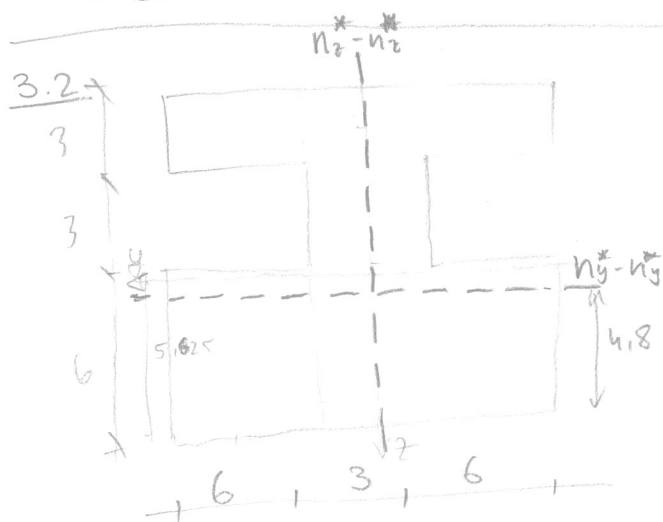
$$\sigma_{кр} = 203,77 \text{ МПа}$$

$$P_{кр} = 203,77 \cdot 10^6 \cdot 114 \cdot 10^{-4} = 2934,29 \text{ КН}$$

$$n_s \geq 3,5$$

$$\frac{P_{кр}}{S_2} \geq 3,5 \quad \frac{2934,29}{0,17829P} \geq 3,5$$

$$P_2 \leq 4676,04 \text{ КН}$$



$$A = 114$$

$$\frac{A}{2} = 72$$

$$W_y^* = 15 \cdot 4,8 \cdot 2,4 + 15 \cdot 1,2 \cdot 0,6 + 3 \cdot 3 \cdot 2,7 + 15 \cdot 3 \cdot 5,7$$

$$W_y^* = 464,4 \text{ см}^3$$

$$W_y = \frac{I_y}{z_{max}} = \frac{2031,75}{6,375} = 318,71 \text{ см}^3$$

$$f_y = \frac{W_y^*}{W_y} = \frac{464,4}{318,71} = 1,457$$

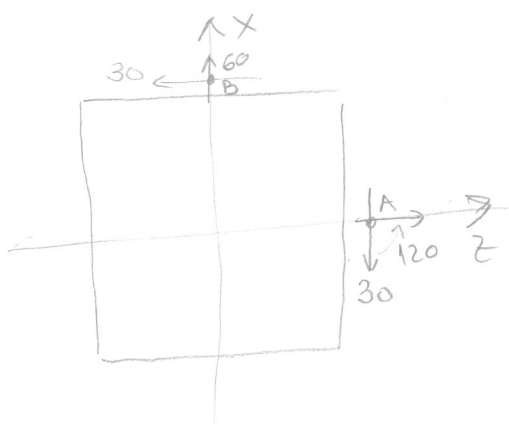
$$W_z^* = ((6 \cdot 6 + 3 \cdot 6) \cdot 4,5 + 15 \cdot 1,2 \cdot 0,75) \cdot 2$$

$$= 513 \text{ см}^3$$

$$W_z = \frac{I_z}{y_{max}} = \frac{2538}{7,5} = 338,4$$

$$f_z = \frac{W_z^*}{W_z} = \frac{513}{338,4} = 1,515$$

2.1



$$A(120, 30)$$

$$B(60, -30)$$

→ МОРОЗ КРУГ (ДИАМЕТР 20М)

ТЕНЗОР

$$\begin{bmatrix} 60 & 0 & 30 \\ 0 & 0 & 0 \\ 30 & 0 & 120 \end{bmatrix}$$

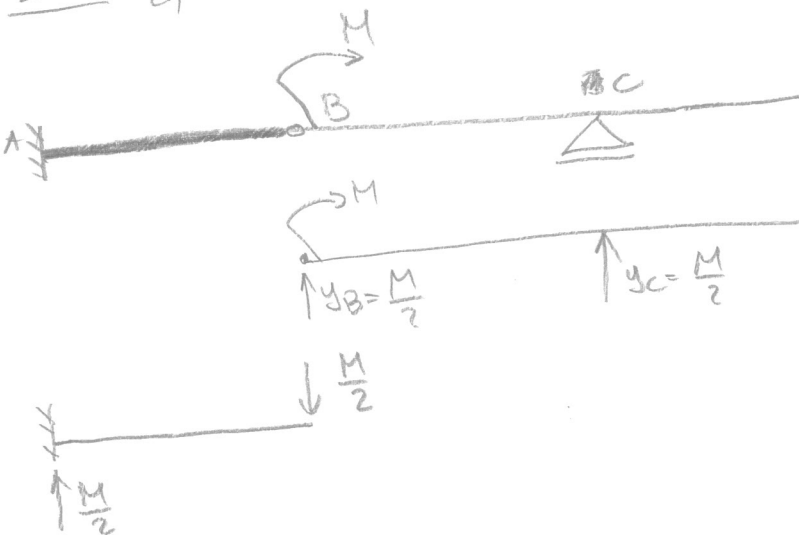
АНАЛИТИЧЕСКИ:

$$\sigma_{1,2} = \frac{\sigma_z + \sigma_x}{2} \pm \sqrt{\frac{(\sigma_z - \sigma_x)^2}{2} + \tau_{zx}^2}$$

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2}$$

$$EI = 200 \text{ KNm}^2$$

2.2 а)



$$\begin{aligned} \sum M_B &= 0 \\ -y_C \cdot 2 + M &= 0 \\ y_C &= \frac{M}{2} \\ y_B &= \frac{M}{2} \end{aligned}$$



$$\begin{aligned} \phi_1 &= 1,125 M \\ \phi_2 &= M \end{aligned}$$

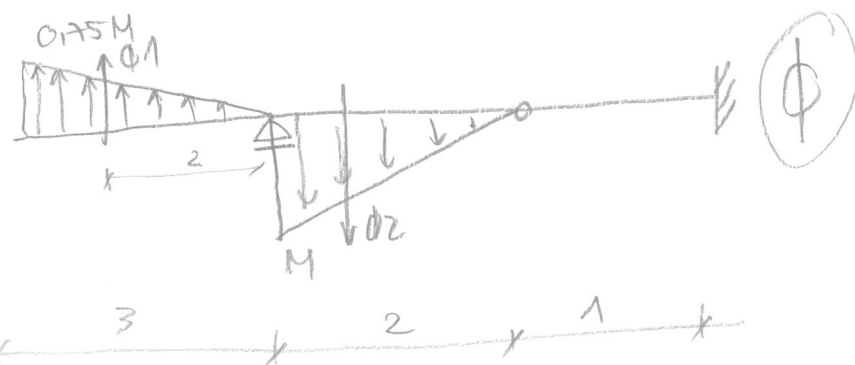
$$\bar{M}_B = \phi_1 \cdot 2 = 2,25 M$$

$$W_B \leq 2 \text{ mm} = 0,002 \text{ m}$$

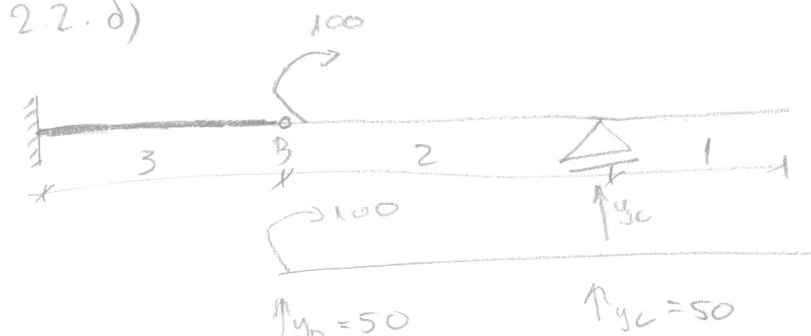
$$W_B = \frac{2,25 M}{200 \cdot 10^3} \leq 0,002$$

$$2,25 M \leq 400$$

$$M \leq 177,78 \text{ Nm}$$



2.2. d)

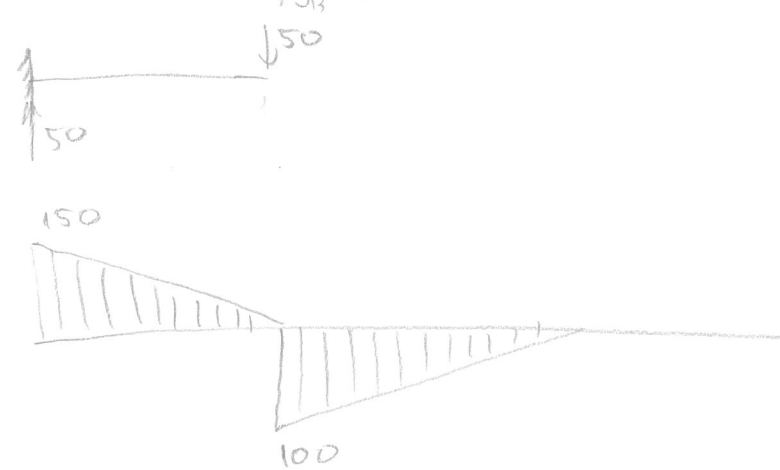


$$\sum M_R = 0$$

$$y_C \cdot 2 - 100 = 0$$

$$y_C = 50$$

$$y_B = 50$$



$$\sigma_{max} = 12 \text{ MPa}$$

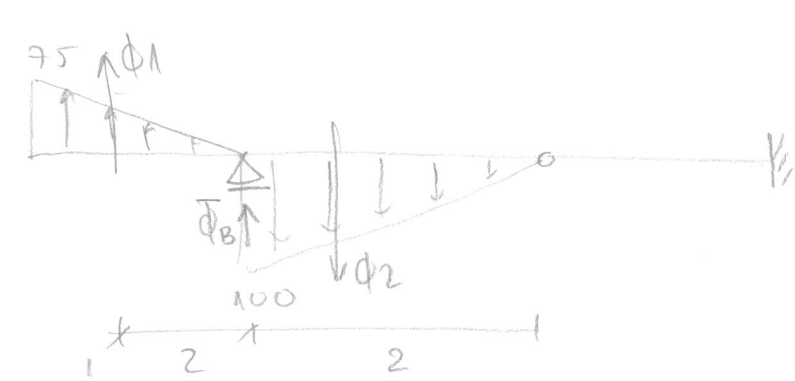
$$M_{max} = 150 \text{ Nm}$$

$$W = \frac{1}{2} R^3 \pi$$

$$\frac{150}{\frac{1}{2} R^3 \pi} \leq 12000000$$

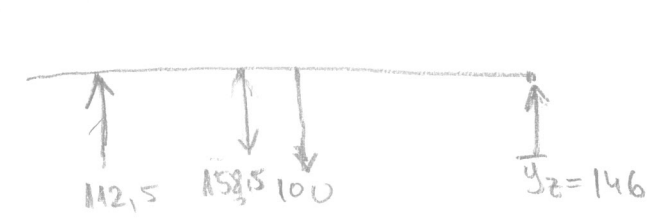
$$R^3 \geq 0,009 \text{ m}$$

$$R \geq 9 \text{ mm}$$



$$\phi_1 = 112,5 \text{ Nm}$$

$$\phi_2 = 100 \text{ N}$$



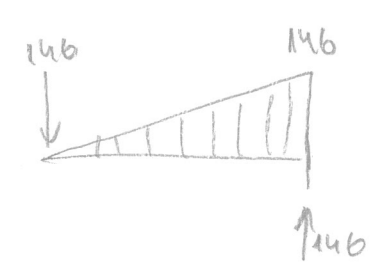
$$\sum \bar{M}_z = 0$$

$$112,5 \cdot 4 - 100 \cdot 1,33 + \phi_B \cdot 2 = 0$$

$$317 = -2 \phi_B$$

$$\phi_B = 158,5 \text{ Nm}$$

$$\bar{y}_2 = 146$$



$$w_D = \frac{146}{200 \cdot 10^3} = 0,00073$$

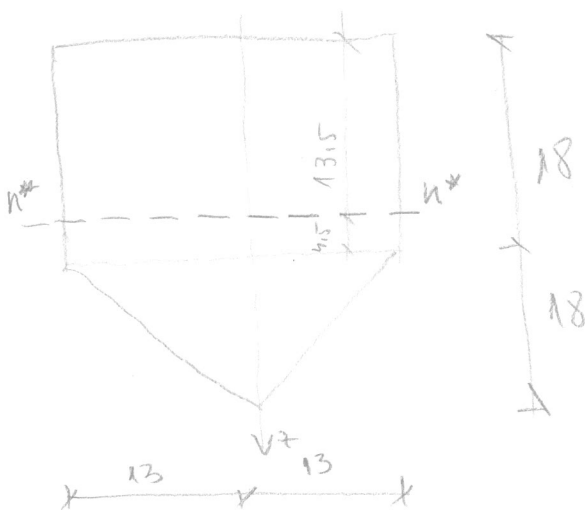
$$= 0,73 \text{ mm}$$

1. а) не знам  
б) анара

$$M_{max}^{AB} = 26,81P = M_{AB}^*$$

$$M_{max}^{BC} = 16,5P = M_{BC}^*$$

$$M_{max}^{CD} = 18P = M_{CD}^*$$



$$A = 702$$

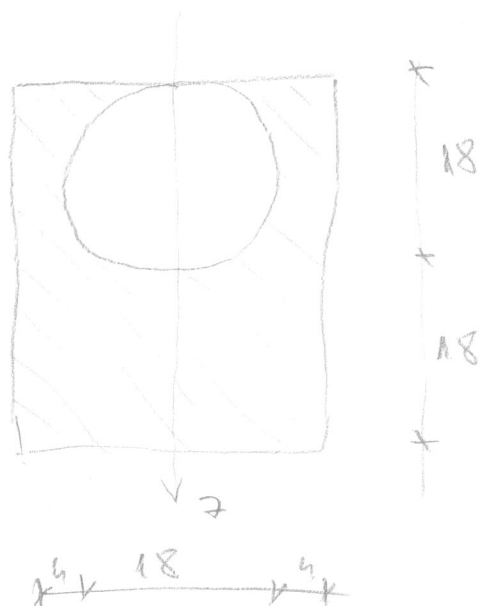
$$\frac{A}{2} = 351$$

$$S_1 = 13,5 \cdot 26 \cdot 6,75 = 2369,25$$

$$S_2 = 4,5 \cdot 26 \cdot 2,125 = 248,625$$

$$S_3 = \frac{18 \cdot 26 \cdot 10,5}{2} = 2457$$

$$W_1^* = 5074,875 \text{ cm}^3$$



$$A = 936 - 254,47 = 681,53 \text{ cm}^2$$

$$\frac{A}{2} = 340,765 \text{ cm}^2$$

$$S_1 = 13,11 \cdot 26 \cdot 6,555 = 2234,34 \text{ cm}^3$$

$$S_2 = 2,445 \cdot 26 \cdot 4,184 = 310,86 \text{ cm}^3$$

$$S_3 = (18 \cdot 26 - 254,47) \cdot 13,89 = 2965,93 \text{ cm}^3$$

$$W_2^* = 5511,13$$

$$W_3^* = \frac{4}{3} R^3 = \frac{4}{3} 18^3 = 7776 \text{ cm}^3$$

$$M_1^* = E_T \cdot W_1^* = 24 \cdot 10^6 \cdot 5074,875 \cdot 10^{-6} = 121,8 \text{ kNm}$$

$$M_2^* = E_T \cdot W_2^* = 24 \cdot 10^6 \cdot 5511,13 \cdot 10^{-6} = 132,27 \text{ kNm}$$

$$M_3^* = 24 \cdot 10^6 \cdot 7776 \cdot 10^{-6} = 186,62 \text{ kNm}$$

распределение напряжений  $\rightarrow$  найдем момент  $\rightarrow$  найдем  $W^*$

$$26,81P = 186,62 \rightarrow P = 6,96$$

$$16,5P = 121,8 \rightarrow P = 7,38$$

$$18P = 132,27 \rightarrow P = 7,35$$

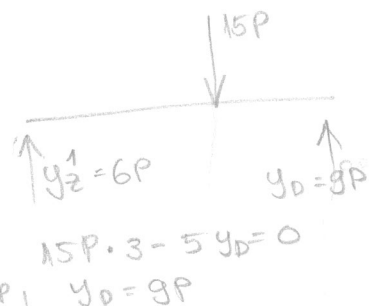
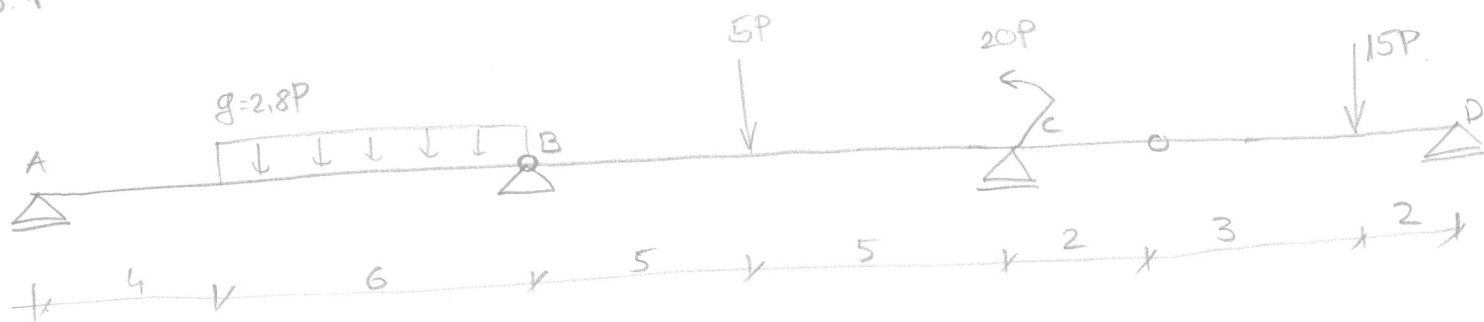
AB  $\rightarrow$  ○

CD  $\rightarrow$  □

BC  $\rightarrow$  □

ВАЖНО ОБАКО,  
ПРОВЕРИТИ КОМБИНАЦИЈЕ

3.1

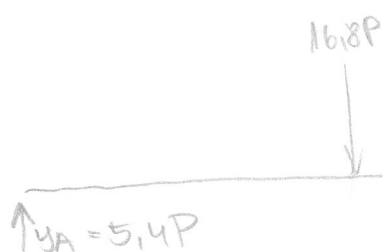


$$6P \cdot 12 - 20P - y_c \cdot 10 + 5P \cdot 5 = 0$$

$$72P - 20P - 10y_c + 25P = 0$$

$$y_c = 7.7P$$

$$y_z^2 = 3.3P$$



$$y_A \cdot 10 - 16.8P \cdot 3 = 0$$

$$10y_A = 50.4P$$

$$y_A = 5.4P$$

$$y_z^2L = 11.4P$$

$$y_B = y_z^2L + y_z^2D = 14.7P$$

